Harmonic maps into Busemann nonpositive curvature spaces

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Abstract

We are concerned with a harmonic map into a Busemann nonpositive curvature space from a finitely generated group acting on the space by isometries. The Busemann nonposive curvature spaces are characterized by the property that the distance between all pairs of parametrized geodeics is convex. A Riemannian manifold with nonpositive curvature and strictly convex Banach space are examples of a Busemann nonpositive curvature space.

We show that if there exists a constant such that the square of the norm of the gradient not less than the constant times the energy on a set of equivariant maps into a Busemann nonpositive curvture space, then the harmonic map into an ultralimit of the Busemann nonpositive curvature space is constant. On the other hand, for a family of Busemann nonpositive curvature space if the harmonic maps are constant, then the inequality between the energy and the gradient holds for any Busemann nonpositive curvature spaces in the family.